APPLICATION OF POD AND PREDICTIVE CONTROL TECHNIQUES TO THE CONTROL OF THE TEMPERATURE PROFILE OF AN ONE-DIMENSIONAL BAR

Oscar Mauricio Agudelo*, Jairo José espinosa†, Bart De Moor*

*Research Group SCD-SISTA, Electrical Engineering Department, Katholieke Universiteit Leuven
Kasteelpark Arenberg 10,
B-3001 Heverlee (Leuven), Belgium

†Facultad de Minas, Universidad Nacional de Colombia
Carrera 80 No. 65-223
Medellín, Colombia

Emails: mauricio.agudelo@esat.kuleuven.be, jairo.espinosa@ipcos.com, bart.demoor@esat.kuleuven.be

Abstract— The simulation and control of PDE's (Partial differential Equations) have been difficult due to the high order of the discretized systems used to approach their solutions. Recently the use of POD (Proper Orthogonal Decomposition) has opened the doors to a simplification making the design of the control systems more tractable. This paper presents the results of applying POD and predictive control techniques to the control of the temperature profile of an one-dimensional bar, whose dynamics is modeled by a parabolic PDE.

Keywords— Proper Orthogonal Decomposition, model reduction, heat transfer process, predictive control.

Resumen— La simulación y el control de sistemas descritos por ecuaciones diferenciales parciales suelen ser tareas complicadas ya que los sistemas discretos que son usados para aproximar dichas ecuaciones poseen ordenes muy elevados. Recientemente el uso de una técnica conocida como POD (Proper Orthogonal Decomposition) ha permitido obtener modelos de orden reducido que hacen posible el diseño de sistemas de control para este tipo de procesos. Este articulo presenta los resultados obtenidos al aplicar POD y Control Predictivo en el control del perfil de temperatura de una barra de una dimensión, cuya dinámica es descrita por una ecuación diferencial parcial de tipo parabólico.

Palabras-Clave— Proper orthogonal decomposition, Reducción de modelos, Transferencia de Calor, Control Predictivo.

1 Introduction

Heat transfer is present in almost every kind of physical process and in fact can be the limiting factor for many of them. The heat transfer by conduction is modeled by parabolic Partial Differential Equations (PDE's). These PDE's are usually simulated by discretizing their spatial and temporal domain. As a general rule, the finer the discretization, the more accurate the numerical solution of the PDE's will be. Nevertheless, a fine discretization produces high order models which are not adequate for control design. Therefore a reduced order model of the discretized system has to be found.

Proper Orthogonal Decomposition (POD) is a technique that has been applied in many physical systems governed by PDE's for deriving reduced order models. The advantage of applying this technique is the incorporation of simulation or experimental data as well as the existing physical relationships (when the Galerkin projection is used) from the original model (Hazenberg et al., 2003).

This paper presents the application of POD and Predictive control techniques to the control of the temperature profile of an one-dimensional bar. Initially an MPC controller without a disturbance model was designed. The control objective was to allow the bar to reach a desired temperature distribution in steady state as fast as possible, satisfying at the same time the process constraints. Afterwards, an MPC with a disturbance model was implemented in order to reject the perturbations that affect the bar. Both MPCs were based on the reduced order model of the system which was derived by means of the POD technique.

The paper presents a description of the process and its discretization in section 2. Section 3 shows the derivation of the reduced order model of the bar by means of POD. In section 4 the design of the MPC controllers is described. Besides the simulation results are presented. Finally section 5 draws the main conclusions.

2 One-Dimensional Bar

The system to be controlled is a "perfectly insulated heated bar" (see figure 1), which has 3 inputs ($u_1(t)$,$u_2(t)$ and $u_3(t)$) and 2 measurement points ($y_1(t)$ and $y_2(t)$). It's important to underline that the inputs of the bar correspond to the boundary conditions of the PDE describing the system.

If only temperature variations in the $x-$direction are considered, the dynamics of the temperature $T(x,t)$ of the bar can be modeled by the following PDE :

$$G\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\partial T(x,t)}{\partial t} \quad , \quad G = \frac{\lambda}{\rho c_p} \qquad (1)$$

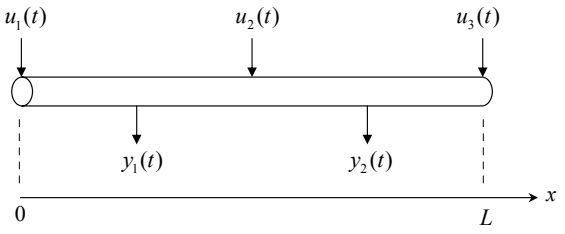

Figure 1: Heated bar. The signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are the boundary conditions at $x = 0$, $x = L/2$ and $x = L$. The measured outputs $y_1(t)$ and $y_2(t)$ are the temperatures of the bar at $x = L/4$ and $x = 3L/4$ respectively.

where ρ is the density of the bar in [Kg m^{-3}], λ is the thermal conductivity in [J s^{-1} m^{-1} K^{-1}], c_p is the heat capacity in [J kg^{-1} K^{-1}] and x is the position in [m].

The initial and boundary conditions of (1) are given by: $T(x,0) = T_0(x)$, $T(0,t) = u_1(t)$, $T(L/2,t) = u_2(t)$ and $T(L,t) = u_3(t)$.

The length of the bar is 0.1 m and the G parameter is equal to 10^{-5}. The initial temperature distribution is set to $T_0(x) = 0°C$ and the input signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ must be between $0°C$ and $150°C$ (input constraints).

2.1 Discretization of the PDE

For design and simulation purposes, equation (1) was discretized by means of the "Implicit Backward Euler method" (a finite difference method) which guarantees that the numerical solution of the PDE will always converge (Yang et al., 2005). In this method, the second partial derivative with respect to x is replaced by a central difference approximation, and the time derivative by a backward difference approximation as follows:

$$G\frac{T_{j+1}^k - T_j^k + T_{j-1}^k}{\Delta x^2} = \frac{T_j^k - T_j^{k-1}}{\Delta t} \qquad (2)$$

$$for\ j = 1, 2, \ldots, P-1, P+1, \ldots, M-1$$

$$for\ k = 1, 2, 3, \ldots, N$$

where $T_j^k = T(x_j, t_k)$ is the temperature of the point $x_j = j\Delta x$ at the time $t_k = k\Delta t$, M is the number of sections in which the bar was divided, Δx is the length of each section, P is the point where $u_2(t)$ is located ($x = L/2$), N is the number of time steps and Δt is the sampling time.

If we define $\mathbf{T}(k)$ as the vector containing the temperatures of all the grid points x_j at each time step k, equation (2) can be cast into a recursive linear system of equations as follows:

$$\mathbf{AT}(k) = \mathbf{T}(k-1) + \mathbf{Bu}(k)$$

$$\mathbf{T}(k+1) = \mathbf{A}^{-1}\mathbf{T}(k) + \mathbf{A}^{-1}\mathbf{Bu}(k+1) \qquad (3)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the matrices describing the system, and $\mathbf{u}(k)$ is the vector of the inputs. The sampling time Δt was set to 1 sec. The spatial domain was divided into $M = 400$ sections which means that the system has 398 states ($\mathbf{T}(k) \in \Re^{398}$). Since it is not easy to design a control system that can deal with 398 states, a reduced-order model of the bar must be found. In this paper such reduced order model is derived using a POD approach.

3 Model Reduction Using POD

In proper orthogonal decomposition, we postulate that $\mathbf{T}(k) \in \Re^q$ can be expanded as a sum of orthonormal basis functions (or spectral form):

$$\mathbf{T}(k) = \sum_{i=1}^{q} a_i(k)\varphi_i(x_d) \qquad (4)$$

$$\varphi_i(x_d) \in \Re^q, \quad a_i(k) \in \Re$$

where $\{\varphi_i(x_d)\}_{i=1}^q$ is a set of orthonormal basis functions (POD basis functions) in the discretized spatial domain x_d, and $\{a_i(k)\}_{i=1}^q$ are the time varying coefficients or POD coefficients associated to each basis function. These POD basis functions are ordered according to their relevance to $\mathbf{T}$(k).

The main dynamics of the system can be represented using the first n most relevant basis functions, since $\{\varphi_i(x_d)\}_{i=1}^n$ condensates the main spatial correlations. An *nth* order approximation of (4) is then given by means of the truncated sequence

$$\mathbf{T}_n(k) = \sum_{i=1}^{n} a_i(k)\varphi_i(x_d), \quad n << q. \qquad (5)$$

An approximate (reduced order) model of the process variable $\mathbf{T}(k)$ can be derived by building a model for the first n time varying coefficients. "This is the essence of model reduction by POD".

The POD basis Functions are determined from simulation or experimental data (Snapshot matrix) of the process. The dynamic model for the first n time varying coefficients can be found by means of the Galerkin projection (Astrid, 2004) or using subspace identification techniques (Huisman, 2005).

The derivation of a reduced order model of (3) was done in 5 steps. These steps are described in the following subsections.

3.1 Generation of the snapshot Matrix

A snapshop matrix $\mathbf{T}_{snap} \in \Re^{398x500}$ was constructed from the system response when PRBNS signals were applied simultaneously to the inputs $u_1(k)$, $u_2(k)$, and $u_3(k)$ of the discrete model of the bar.

$$\mathbf{T}_{snap} = (\mathbf{T}(k=1), \ldots, \mathbf{T}(k=500)) \qquad (6)$$

Along the simulations, a switching probability of 2% was set to the PBRNS signals and 500 samples were collected using a sampling time of 1 sec.

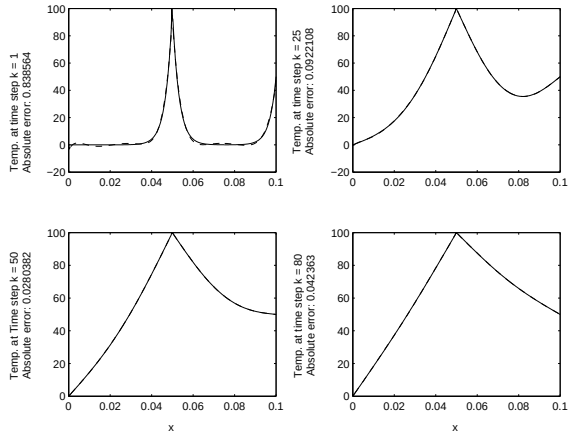

Figure 2: Temperature profile at different time steps. Solid line - full order Model. Dashed line - reduced order model.

3.2 Derivation of the POD Basis Functions

They were derived by calculating the singular value decomposition (SVD) of $\mathbf{T}_{snap}$,

$$\mathbf{T}_{snap} = \mathbf{\Phi}\mathbf{\Sigma}\mathbf{\Psi}^T$$

where $\mathbf{\Phi} \in \Re^{398x398}$ and $\mathbf{\Psi} \in \Re^{500x500}$ are unitary matrices, and $\mathbf{\Sigma} \in R^{398x500}$ is a matrix which contains the singular values of $\mathbf{T}_{snap}$ in a decreasing order on its main diagonal.

The columns of Φ are the POD basis functions.

$$\mathbf{\Phi} \in R^{398x398} = (\varphi_1, \varphi_2, \dots \varphi_{398})$$

3.3 Selection of the most relevant POD basis functions

It was done by checking the singular values of $\mathbf{T}_{snap}$. The larger the singular value the more relevant the basis function is.

The first 10 basis functions associated to the first 10 largest singular values were selected. The 10th order approximation of $\mathbf{T}(k)$ is given by

$$\mathbf{T}_n(k) = \sum_{i=1}^{10} a_i(k)\varphi_i(x_d) = \mathbf{\Phi}_n\mathbf{a}(k) \qquad (7)$$

where $\mathbf{\Phi}_n = (\varphi_1, \varphi_2, \dots \varphi_{10})$ and $\mathbf{a}(k) = (a_1(k), a_2(k), \dots, a_{10}(k))^T$.

3.4 Construction of the model for the first n=10 POD coefficients

As it was mention before, the model for the POD coefficients can be derived by means of subspace identification techniques (Huisman, 2005) or by the Galerkin projection (Astrid, 2004). Although these two approaches were tested successfully (they gave quite similar results), in this paper only the Galerkin projection approach is discussed.

If we define a residual function $R(\mathbf{T})$ for equation (3) as follows:

$$R(\mathbf{T}) = \mathbf{T}(k+1) - \mathbf{A}^{-1}\mathbf{T}(k) - \mathbf{A}^{-1}\mathbf{B}\mathbf{u}(k+1) \quad (8)$$

and we replace $\mathbf{T}(k)$ by its nth order approximation $\mathbf{T}_n(k)$ in (8), the Galerkin projection states that the projection of $R(\mathbf{T}_n)$ on the space spanned by the basis functions $\mathbf{\Phi}_n$ vanishes. That is,

$$\langle R(\mathbf{T}_n), \varphi_i(x_d)\rangle = 0; \;\; i = 1, \dots n \qquad (9)$$

where $\langle .,. \rangle$ denotes inner product. Replacing $\mathbf{T}(k)$ by its nth order approximation $\mathbf{T}_n(k) = \mathbf{\Phi}_n\mathbf{a}(k)$ in equation (3), and applying the inner product criterion (Galerkin projection) to the resulting equation we have:

$$\langle \mathbf{\Phi}_n\mathbf{a}(k+1), \mathbf{\Phi}_n\rangle = \left\langle \mathbf{A}^{-1}\mathbf{\Phi}_n\mathbf{a}(k) + \mathbf{A}^{-1}\mathbf{B}\mathbf{u}(k+1), \mathbf{\Phi}_n \right\rangle. \quad (10)$$

By evaluating the inner product in (10),

$$\mathbf{\Phi}_n^T\mathbf{\Phi}_n\mathbf{a}(k+1) = \mathbf{\Phi}_n^T\mathbf{A}^{-1}\mathbf{\Phi}_n\mathbf{a}(k) + \mathbf{\Phi}_n^T\mathbf{A}^{-1}\mathbf{B}\mathbf{u}(k+1)$$

$$\mathbf{a}(k+1) = \mathbf{\Phi}_n^T\mathbf{A}^{-1}\mathbf{\Phi}_n\mathbf{a}(k) + \mathbf{\Phi}_n^T\mathbf{A}^{-1}\mathbf{B}\mathbf{u}(k+1) \qquad (11)$$

we obtain the model for the first n time varying coefficients. Equation (11) is not in the standard state space representation because of the term $\mathbf{u}(k+1)$. However from its simulation diagram we can recast (11) into a standard representation and obtain the reduced order model of the bar with only 10 states,

$$\begin{aligned} \mathbf{x}(k+1) &= \tilde{\mathbf{A}}\mathbf{x}(k) + \tilde{\mathbf{B}}\mathbf{u}(k) \\ \mathbf{a}(k) &= \tilde{\mathbf{C}}\mathbf{x}(k) + \tilde{\mathbf{D}}\mathbf{u}(k) \\ \mathbf{T}_n(k) &= \mathbf{\Phi}_n\mathbf{a}(k) \end{aligned} \qquad (12)$$

where $\tilde{\mathbf{A}} = \tilde{\mathbf{C}} = \mathbf{\Phi}_n^T\mathbf{A}^{-1}\mathbf{\Phi}_n$ and $\tilde{\mathbf{B}} = \tilde{\mathbf{D}} = \mathbf{\Phi}_n^T\mathbf{A}^{-1}\mathbf{B}$.

3.5 Validation of the reduced order model

For validating the reduced order model, constant inputs $u_1(k) = 0°C$, $u_2(k) = 100°C$ and $u_3(k) = 50°C$ were applied to the full order model and to the reduced order model, and afterwards their outputs were compared. Figure 2 shows the temperature profile of the bar at the time steps $k = 1$, $k = 25$, $k = 50$ and $k = 80$ for each model. It is really difficult to observe differences between the responses of the models. In order to measure the quality of the reduced order model the average of the absolute error was calculated by means of the following formula:

$$\varepsilon = \frac{1}{N_s}\sum_{k=1}^{N_s} |\mathbf{T}(k) - \mathbf{T}_n(k)|$$

where $N_s = 100$ is the number of time steps. The the maximum value for the error was only 0.18 degrees Celsius, which means that the reduced order model with only 10 states approximates very well the behavior of the full order model (398 states).

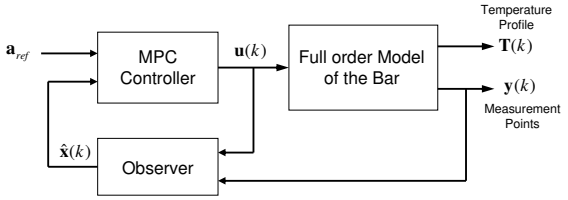

Figure 3: MPC control scheme without a disturbance model.

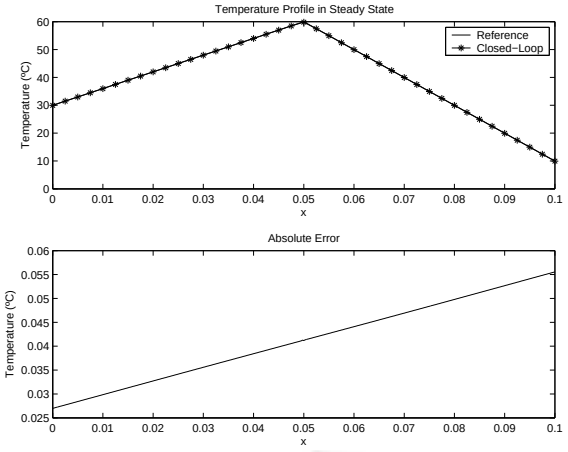

Figure 4: steady state Temperature profile obtained with the MPC control scheme.

4 Control Scheme

The control goal is to allow the bar to reach a desired temperature distribution in steady state as fast as possible. In addition, the control actions must satisfy the inputs constraints of the process ($0°C \leq u_1(k) \leq 150°C$, $0°C \leq u_2(k) \leq 150°C$ and $0°C \leq u_3(k) \leq 150°C$). In the top plot of the figure 4 the desired temperature profile $\mathbf{T}_{ref}$ for the bar can be observed. This temperature profile corresponds to the steady state temperature distribution when the bar is heated from zero temperature by constant inputs $u_1(k) = 30°C$, $u_2(k) = 60°C$ and $u_3(k) = 10°C$. Since $\mathbf{T}_{ref}$ can be expressed in terms of the POD coefficients as follows:

$$\mathbf{r} = \mathbf{\Phi}^T \mathbf{T}_{ref} \tag{13}$$

we can control the temperature profile of the bar by controlling the POD coefficients. The reference vector for the first 10 POD coefficients is denoted by $\mathbf{a}_{ref}$ and is equal to the 10 first elements of the vector $\mathbf{r}$ in (13). For controlling the first 10 POD coefficients and consequently the temperature profile of the bar, an MPC control scheme was designed (see figure 3). In this scheme the MPC controller uses the reduced order model given by (12) to predict the future behavior of the process. An observer, which in this case is a Kalman Filter, is used for estimating the state of the reduced order model from the measurements $\mathbf{y}(k) = (y_1(k), y_2(k))^T$ and the process inputs $\mathbf{u}(k) = (u_1(k), u_2(k), u_3(k))^T$. The observer equation is given by,

$$\hat{\mathbf{x}}(k+1) = \tilde{\mathbf{A}}\hat{\mathbf{x}}(k) + \tilde{\mathbf{B}}\mathbf{u}(k) + \mathbf{L}\left(\mathbf{y}(k) - \hat{\mathbf{y}}(k)\right) \tag{14}$$

$$\hat{\mathbf{a}}(k) = \tilde{\mathbf{C}}\hat{\mathbf{x}}(k) + \tilde{\mathbf{D}}\mathbf{u}(k)$$

$$\hat{\mathbf{y}}(k) = \mathbf{C}_s \hat{\mathbf{T}}_n(k) = \mathbf{C}_s \mathbf{\Phi}_n \hat{\mathbf{a}}(k)$$

where $\hat{\mathbf{x}}(k)$ is the estimated state, $\hat{\mathbf{y}}(k)$ is the estimated output vector, $\mathbf{L}$ is the observer gain (Kalman gain) and $\mathbf{C}_s$ is a selection matrix which selects the measured temperatures ($y_1(k)$ and $y_2(k)$) from the vector $\mathbf{T}_n(k)$. The estimated state $\hat{\mathbf{x}}(k)$ is used together with the reference vector $\mathbf{a}_{ref}$ by the MPC controller to calculate the appropriated control actions to be applied to the bar. The minimization problem that is solved by the MPC controller is defined as:

$$\min_{a,\Delta u} J = \sum_{k=1}^{N_p} \left(\mathbf{a}_{ref}(k) - \mathbf{a}(k)\right)^T \mathbf{Q} \left(\mathbf{a}_{ref}(k) - \mathbf{a}(k)\right) + \sum_{k=0}^{Nc-1} \mathbf{\Delta u}(k)^T \mathbf{R} \mathbf{\Delta u}(k) \tag{15}$$

subject to

$$\mathbf{x}(k+1) = \tilde{\mathbf{A}}\mathbf{x}(k) + \tilde{\mathbf{B}}\mathbf{u}(k)$$

$$\mathbf{a}(k) = \tilde{\mathbf{C}}\mathbf{x}(k) + \tilde{\mathbf{D}}\mathbf{u}(k)$$

$$\mathbf{u}_{\min} \leq \mathbf{u}(k) \leq \mathbf{u}_{\max}$$

where $\Delta\mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$, N_p is the prediction horizon, N_c is the control horizon, $\mathbf{Q}$ and $\mathbf{R}$ are positive definite weighting matrices, and $\mathbf{u}_{min}$ and $\mathbf{u}_{max}$ are the lower and upper bounds of $\mathbf{u}(k)$.

The control horizon N_c was set to 7 samples and the prediction horizon N_p was selected according to the following criterion: "Prediction Horizon = Control Horizon + Largest Settling Time = 80 samples". $\mathbf{u}_{min}$ and $\mathbf{u}_{max}$ were selected according to the input constraints of the process and the weighting matrices in this way: $\mathbf{Q} = \mathbf{I}_{10x10}$, $\mathbf{R} = \mathbf{I}_{3x3}$.

The simulation results are shown in figures 4 and 5. In Figure 4 we can observe the steady state temperature profile of the bar when the MPC control scheme is used. Also the absolute error between the reference and the steady state response of the closed loop system is shown. The maximum deviation in the temperature profile is only $0.055°C$. Figure 5 presents the evolution in time of the temperature at the measurement points and the control actions of the MPC controller, which satisfy the input constraints of the process. The closed-loop responses reach steady state faster than the open-loop responses because the control system pushes the inputs to the limits. In spite of the dramatic reduction of model order, on which the controller is based, the controller performs very well.

Let's suppose that the left edge of the bar is exposed to the ambient temperature (20°C) from a specific time. It implies that temperature at the left edge would be given by the ambient temperature plus $u_1(k)$. Figure 6 presents the steady state temperature profile of the bar in disturbed conditions with the MPC controller. It is clear that the behavior of the control system in the presence of input disturbances is not good. At the left edge of the bar there is a large temperature deviation of $14.22°C$. This was an expected result, because the control scheme does not include a mechanism for rejecting such kind of disturbances. So, in order to overcome such limitation, an MPC controller with a disturbance model was designed.

4.1 MPC controller with a Disturbance Model

The formulation of the MPC controller with a disturbance model is given by

$$\min_{a,\Delta u} J = \sum_{k=1}^{N_p} \left(\mathbf{a}_{ref}(k) - \mathbf{a}(k)\right)^T \mathbf{Q} \left(\mathbf{a}_{ref}(k) - \mathbf{a}(k)\right) + \sum_{k=0}^{Nc-1} \mathbf{\Delta u}(k)^T \mathbf{R} \mathbf{\Delta u}(k) \qquad (16)$$

subject to

$$\mathbf{x}(k+1) = \tilde{\mathbf{A}}\mathbf{x}(k) + \tilde{\mathbf{B}}\mathbf{u}(k) + \mathbf{F}\mathbf{d}(k)$$

$$\mathbf{a}(k) = \tilde{\mathbf{C}}\mathbf{x}(k) + \tilde{\mathbf{D}}\mathbf{u}(k) + \mathbf{W}\mathbf{d}(k)$$

$$\mathbf{d}(k+1) = \mathbf{d}(k), \qquad \mathbf{u}_{\min} \leq \mathbf{u}(k) \leq \mathbf{u}_{\max}$$

where $\Delta\mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$, N_p is the prediction horizon, N_c is the control horizon, $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices, $\mathbf{u}_{min}$ and $\mathbf{u}_{max}$ are the lower and upper bounds of $\mathbf{u}(k)$, $\mathbf{F}$ and $\mathbf{W}$ are the disturbance model matrices and $\mathbf{d}(k)$ is the disturbance vector. Since the disturbance $\mathbf{d}(k)$ and the state $\mathbf{x}(k)$ are unknown, they must be estimated by means of a new observer with the following formulation:

$$\begin{pmatrix} \hat{\mathbf{x}}(k+1) \\ \hat{\mathbf{d}}(k+1) \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{A}} & \mathbf{F} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}}(k) \\ \hat{\mathbf{d}}(k) \end{pmatrix} + \begin{pmatrix} \tilde{\mathbf{B}} \\ \mathbf{0} \end{pmatrix} \mathbf{u}(k) + \begin{pmatrix} \mathbf{L}_x \\ \mathbf{L}_d \end{pmatrix} \left(\mathbf{y}(k) - \hat{\mathbf{y}}(k)\right) \qquad (17)$$

$$\hat{\mathbf{a}}(k) = \begin{pmatrix} \tilde{\mathbf{C}} & \mathbf{W} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}}(k) \\ \hat{\mathbf{d}}(k) \end{pmatrix} + \tilde{\mathbf{D}}\mathbf{u}(k)$$

$$\hat{\mathbf{y}}(k) = \mathbf{C}_s \hat{\mathbf{T}}_n(k) = \mathbf{C}_s \mathbf{\Phi}_n \hat{\mathbf{a}}(k)$$

where $\hat{\mathbf{x}}(k)$ is the estimated state, $\hat{\mathbf{d}}(k)$ is the estimated disturbance vector, $\hat{\mathbf{y}}(k)$ is the estimated output vector, $(\mathbf{L}_x \quad \mathbf{L}_d)^T$ is the observer gain (Kalman gain) and $\mathbf{C}_s$ is a selection matrix which selects the measured temperatures ($y_1(k)$ and $y_2(k)$) from the vector $\mathbf{T}_n(k)$.

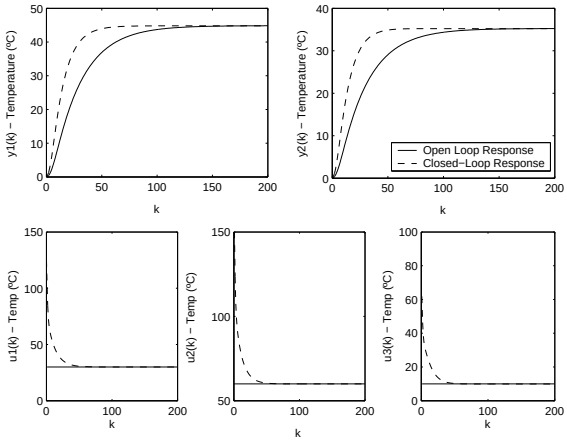

Figure 5: Temperature at the measurement Points and Control Actions of the MPC controller.

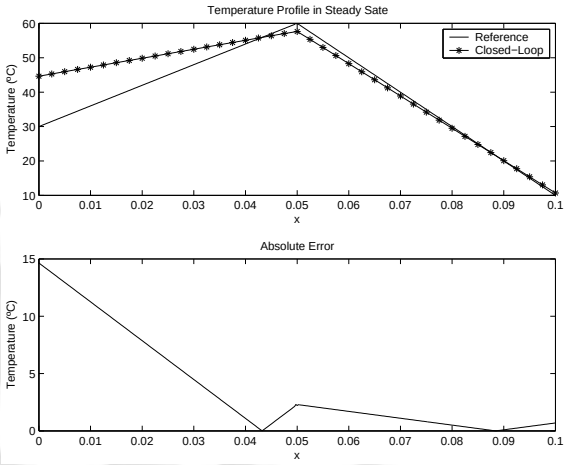

Figure 6: Steady state temperature profile of the bar in disturbed conditions with the MPC controller.

The number of disturbance terms that can be estimated without losing observability is equal to the number of sensors (Pannocchia and Rawlings, 2001). Therefore $\mathbf{d}(k)$ has 2 entries since the bar has only 2 sensors. The disturbance model is defined by the matrices $\mathbf{F}$ and $\mathbf{W}$. Among the infinite combinations of these matrices, two of them have a particular physical interpretation: the Output disturbance Model when $\mathbf{F} = \mathbf{0}$ and $\mathbf{W} = \mathbf{I}$ and the Input disturbance Model when $\mathbf{F} = \tilde{\mathbf{B}}$ and $\mathbf{W} = \tilde{\mathbf{D}}$. Given the nature of the disturbance that is affecting the bar, the Input disturbance model is the most appropriated choice. In this design the disturbances associated to the inputs $u_1(k)$ and $u_2(k)$ were estimated by selecting the disturbance model matrices in the following way: "$\mathbf{W}$ = first two columns of $\tilde{\mathbf{D}}$" and "$\mathbf{F}$ = first two columns of $\tilde{\mathbf{B}}$". The control and prediction horizon were set to 7 samples and 80 samples respectively, $\mathbf{u}_{min}$ and $\mathbf{u}_{max}$ were selected according to the input constraints of the process and the weighting matrices in this way: $\mathbf{Q} = \mathbf{I}_{10x10}$, $\mathbf{R} = \mathbf{I}_{3x3}$.

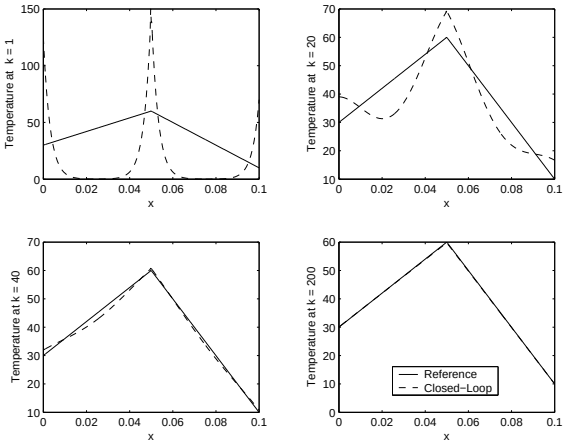

Figure 7: Evolution of the temperature profile before the disturbance (MPC with a disturbance model).

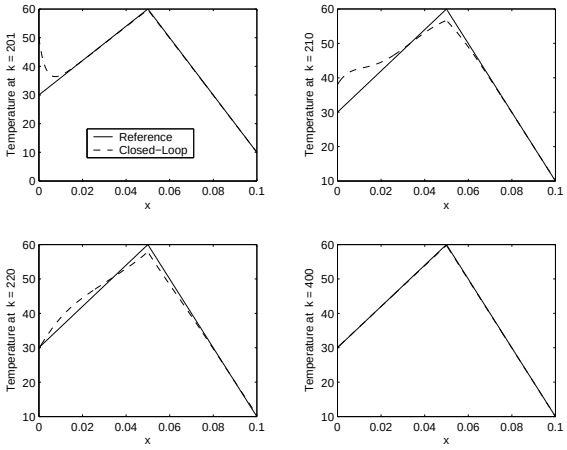

Figure 8: Evolution of the temperature profile after the disturbance (MPC with a disturbance model).

The simulation results of the MPC controller with an input disturbance model are shown in figures 7 and 8. The left edge of the bar was exposed to the ambient temperature at the time step $k = 201$. Figure 7 shows the evolution of the temperature profile from the initial temperature distribution until the system reaches the steady state, just before the disturbance is applied. The evolution of the temperature profile once the disturbance is applied is presented in figure 8. The controller worked very well and rejected the perturbation quickly. In steady state, the maximum deviation in the temperature profile was only $0.21°C$ which is practically negligible. Although it is not shown in this paper, the control actions of the MPC Controller with a disturbance model were all the time within the permitted limits.

5 Conclusions

In this paper the results of applying POD and predictive control techniques to the control of the temperature profile of an one-dimensional bar have been presented. A big model reduction has been achieved by means of the POD technique that has made possible the control design. Initially, an MPC controller without a disturbance model was designed for controlling the temperature distribution when no input disturbances were applied to the bar. Its performance was very good in spite of the dramatic reduction of model order on which the controller is based. Afterwards, an MPC controller with a disturbance model was designed in order to reject the kind of disturbances that affect the bar. This controller worked very well and rejected the perturbations quickly.

Acknowledgments

This research was supported by: • Research Council KUL: GOA AMBioRICS, CoE EF/05/006 Optimization in Engineering, several PhD/postdoc & fellow grants; •Flemish Government: ◦FWO: PhD/postdoc grants, projects, G.0407.02 (support vector machines), G.0197.02 (power islands), G.0141.03 (Identification and cryptography), G.0491.03 (control for intensive care glycemia), G.0120.03 (QIT), G.0452.04 (new quantum algorithms), G.0499.04 (Statistics), G.0211.05 (Nonlinear), G.0226.06 (cooperative systems and optimization), G.0321.06 (Tensors), G.0553.06 (VitamineD), research communities (ICCoS, ANMMM, MLDM); ◦IWT: PhD Grants,GBOU (McKnow), Eureka-Flite2 • Belgian Federal Science Policy Office: IUAP P5/22 ('Dynamical Systems and Control: Computation, Identification and Modelling', 2002-2006) ; PODO-II (CP/40: TMS and Sustainability); •EU: FP5-Quprodis; ERNSI; •Contract Research/agreements: ISMC/IPCOS, Data4s, TML, Elia, LMS, Mastercard. Dr. Bart De Moor is a full professor at the Katholieke Universiteit Leuven, Belgium.

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